CHARACTERISTICS OF SIMULTANEOUS THAWING AND EROSION OF FROZEN SAND

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An ablation model of frozen soil in which the thaw front does not coincide with the breakup front is proposed.

Drilling of ice-cemented frozen sand under conditions of a positive temperature is accompanied by the formation of pockets; this is a result of the simultaneous thawing and erosion of the walls of the well [1]. It was initially assumed that all of the sand in the thawed layer is subject to erosion [2], so that pocket formation was reduced to an ablation process. This representation does not agree with the facts concerning the formation of a clay crust on the well wall, as well as slipping of a large quantity of sand into the well shaft if circulation stops; on the contrary, these processes show that a significant part of the thawed layer remains where it is formed. In this connection it was proposed that pocket formation is a more complicated process combining thawing and erosion in which the boundary where thawing of interstitial ice starts runs ahead of the boundary where breakup of the mineral framework starts, and between those boundaries there forms a transitional layre containing both water and ice. The transitional zone forms owing to the fact that the rate of heat transfer is significantly higher along the mineral framework than along the ice in the pores.

It is shown below that this, more general, ablation process in which the thaw front does not coincide with the breakup front admits a simple quantitative description when in the transitional zone the pore volume freed up when the ice thaws is immediately filled with water flowing from the outside. The results of the numerical modeling are consistent with the known experimental facts.

We shall assume that a semiinfinite flat wall, consisting of uniform sand, in which pores, outside the thaw front $x \geq x_s(t)$ are entirely filled with ice, is subjected to ablation in the sense indicated. In the intermediate zone $x_m(t) \leq x \leq x_s(t)$ the ice content increases monotonically up to unity at $x = x_s(t)$. Because of the assumption made above that the pore space freed up as the ice melt is filled with water flowing in from outside the space the ice content in the transitional zone is uniquely determined by the degree of saturation with water $\sigma(x, t)$. It is assumed to be equal to zero on the thaw boundary $\sigma(x_s, t) = 0$ and at $_m(t)$ it is constant $\sigma_m = \sigma(x_m, t)$. It is assumed that the corresponding ice content $1 - \sigma_m$ is no longer sufficient to withstand, together with the weak mineral cement, the erosion of the framework by the flow of circulating water, moving with a constant velocity along the wall perpendicular to the x-axis.

The ice in the transitional zone occupies the central part of the pores, and the water lies adjacent to the warmer mineral framework. It is assumed here that the temperature of the framework is equal to that of the water, while the temperature of the ice is constant and equal to the melting temperature T_s. The ice melts as a result of heat flow from the mineral framework. In accordance with this picture heat flow in the transitional zone is determined by the equations

$$\nabla\lambda_1\nabla T_1 - q_1 = (c\rho)_1 \frac{\partial T_1}{\partial t}\ ;\ x_m \leqslant x \leqslant x_s, \qquad (1)$$

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$$q_1 = K_1(T_1 - T_s) = m\rho_{\mathrm{i}} L \frac{\partial\sigma}{\partial t},$$

$$\lambda_1 = (1-m)\lambda_{\mathrm{fw}},\ (c\rho)_1 = m\sigma(c\rho)_{\mathrm{w}} + (1-m)(c\rho)_{\mathrm{fw}}. \tag{2}$$

The last equality in Eqs. (2) expresses the intensity of heat transfer between the framework and the ice in terms of the increase in the degree of saturation with water, which complements the degree of saturation of the pore space with ice to unity by virtue of the assumption made above that water flows freely into the transitional zone from outside the zone. According to [3], the heat going into heating of the water is included on the right side of (1). The contribution of the water film to the limiting thermal conductivity is not included on the right side of Eq. (1), since it participates only in the transverse flow of heat. Because of this the coefficient of heat transfer K_1 depends on σ.

In the frozen rock $x_s(t) < x < \infty$ the heat transfer is described by the equation

$$\nabla\lambda\nabla T = c\rho\frac{\partial T}{\partial t}, \tag{3}$$

where

$$\lambda = m\lambda_{\mathrm{i}} + (1-m)\lambda_{\mathrm{fw}},\ c\rho = m(c\rho)_{\mathrm{i}} + (1-m)(c\rho)_{\mathrm{fw}}.$$

We give the following boundary and initial conditions:

$$t=0:\ T=T_i;\ x=\infty:\ T=T_i;\ x=x_s:\ \lambda_1\nabla T_1 = \lambda\nabla T;$$

$$T_1 = T = T_s;\ x = x_m:\ q = -\lambda_1\nabla T_1;\ T_1 = T_m.$$

Here q is the given heat flux at the surface of ablation $x = x_m(t)$ and T_m is the unknown temperature on this surface and is determined together with x_m and x_s in the course of the solution of the problem.

To solve the problem we shall assume that σ_m is small and the dependence of the coefficient K_1 on σ can be neglected.

Using the dimensionless variables and parameters

$$\tau = \alpha_1\beta_1 t;\ \zeta = x\sqrt{\beta_1};\ \eta_s = x_s\sqrt{\beta_1};\ \eta_m = x_m\sqrt{\beta_1};$$

$$\alpha_1 = \lambda_1/(c\rho)_1;\ \beta_1 = K_1/\lambda_1;\ \beta = \alpha_1/\alpha;\ \zeta_m = q/\lambda_1\sqrt{\beta_1}(T_s - T_i);$$

$$\theta_1 = \frac{T_1 - T_s}{T_s - T_i};\ \theta = \frac{T - T_s}{T_s - T_i};\ \theta_m = \frac{T_m - T_s}{T_s - T_i}$$

the problem posed reduces to solving the equations

$$\beta\frac{\partial\theta}{\partial t} = \frac{\partial^2\theta}{\partial\zeta^2}\ (\eta_s < \zeta < \infty);\quad \frac{\partial\theta_1}{\partial\tau} = \frac{\partial^2\theta_1}{\partial\zeta^2} - \theta_1\ (\eta_m < \zeta < \eta_s) \tag{4}$$

with the following initial and boundary conditions:

$$\zeta = \infty:\ \theta = -1;\ \tau = 0:\ \theta = -1;\ \zeta = \eta_s:\ \theta_1 = \theta = 0;$$

$$\lambda_1\frac{\partial\theta_1}{\partial\zeta} = \lambda\frac{\partial\theta}{\partial\zeta};\ \zeta = \eta_m:\ \theta_1 = \theta_m;\ \zeta_m = -\lambda\frac{\partial\theta_1}{\partial\zeta}. \tag{5}$$

We shall seek θ and θ_1 as functions of only one variable $\Delta = \eta - \xi$. Substitution into Eqs. (1)-(3) shows that

$$\theta = \exp\beta\dot{\eta}_s\Delta - 1, \tag{6}$$

$$\theta_1 = \theta_m f_1(\Delta);\ f_1(\Delta) = \frac{\exp\frac{1}{2}\dot{\eta}_s\Delta\,\mathrm{sh}\,\Delta\sqrt{\left(\frac{1}{2}\dot{\eta}_s\right)^2+1}}{\exp\frac{1}{2}\dot{\eta}_s\Delta_m\,\mathrm{sh}\,\Delta_m\sqrt{\left(\frac{1}{2}\dot{\eta}_s\right)^2+1}}, \tag{7}$$

where $\Delta_m = \eta_s - \eta_m$.

The conditions given in Eq. (5) on the boundary temperatures are thus automatically satisfied, and the conditions for continuity of the heat fluxes head to the following equalities:

$$\theta_m f_1'(\Delta_m) = \gamma\dot{\eta}_s;\ \gamma = \lambda\alpha_1/\lambda_1\alpha, \tag{8}$$

$$\zeta_m = \theta_m f_1'(\Delta_m), \tag{9}$$

from which Δ_m and $\dot{\eta}_s$ are determined for given values of θ_m and ζ_m. If the latter quantities are constant, then Δ_m and $\dot{\eta}_s$ are also constant. Then the solution obtained is of the traveling-wave type [4]. Stefan's solution for ablation [5], which can be obtained from our solution by assuming that the transitional zone has zero thickness, is also of the same type.

To determine the unknown θ_m we shall employ the equalities (2) for heat transfer within the transitional zone. These equations have the following dimensionless form:

$$\theta_1 = l\frac{\partial\sigma}{\partial t},\ l = m\rho_i L\alpha_1/\lambda_1(T_s - T_i).$$

It is obvious that the degree of saturation with water σ in the transitional zone is also a function of Δ, $\sigma = \phi(\Delta)$, and according to the last equality

$$\theta_m f_1(\Delta) = \dot{\eta}_s l\varphi'(\Delta).$$

By integration we obtain from here, using the condition $\sigma(x_s, t) = 0$,

$$\varphi(\Delta) = \frac{\theta_m}{l\dot{\eta}_s}\int_0^\Delta f_1(\Delta)\,d\Delta.$$

Since the function $f_1(\Delta)$ from Eq. (7) satisfies the equation

$$f_1(\Delta) = f_1''(\Delta) - \dot{\eta}_s f_1'(\Delta),$$

the integral leads to the equality

$$\sigma_m = \varphi(\Delta_m) = \frac{\theta_m}{\dot{\eta}_s l}[f_1'(\Delta_m) - f'(0)] - \frac{\theta_m}{l}f_1(\Delta_m).$$

Using Eqs. (8) and (9) and $f_1(\Delta_m) = 1$, we obtain

$$\theta_m = \frac{\zeta_m}{\dot{\eta}_s} - \gamma - \sigma_m l. \tag{10}$$

The equalities Eqs. (8), (9), and (10) give a complete system of equations for determing the unknowns θ_m, Δ_m and $\dot{\eta}_s$.

Combining Eq. (10) with Eqs. (8) and (9) in order to eliminate θ_m, we obtain the following two equations:

$$\psi(y) = y + \frac{1}{\operatorname{th} x}, \tag{11}$$

$$\gamma e^{xy}\psi(y) = \zeta_m\frac{\sqrt{1-y^2}}{y}\sqrt{1-\operatorname{th}^2 x}, \tag{12}$$

where

$$x = \Delta_m\sqrt{\left(\frac{1}{2}\dot{\eta}\right)^2 + 1},\ y = \frac{1}{2}\dot{\eta}_s\Big/\sqrt{\left(\frac{1}{2}\dot{\eta}_s\right)^2 + 1},$$

$$\psi(y) = \frac{1}{2y} - \frac{\gamma + \zeta_m l}{\zeta_m\sqrt{1-y^2}}.$$

This system can be easily solved by the method of successive approximations. The following values of the physical parameters were used in the calculation: $\lambda_{fw} = 4.64$ W/(m·K); $\lambda_i = 2.22$ W/(m·K); $(c\rho)_{fw} = 1.63\cdot10^6$ J/(m³·K); $(c\rho)_i = 1.94\cdot10^6$ J/(m³·K); m = 0.4; K_1 = 300 W/(m³·K); q = 10 and 100 W/m².

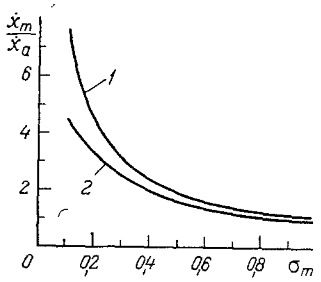

Fig. 1

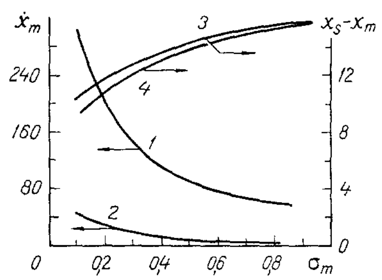

Fig. 2

Fig. 1. Ratio of the rate of erosion $\dot{x}_m$ with different values of σ_m to the rate of ablation in the standard formulation: 1) $T_i = -1°C$; 2) $T_i = -5$.

Fig. 2. Velocity of the erosion front $\dot{x}_m$ (cm/day) and the thickness of the intermediate zone $x_s - x_m$(cm) versus the intensity of the heat flux at the front: 1, 3) $q = 10\ W/m^2$; 2, 4) 100.

It is natural to compare the results of the calculations of $dx_m/dt = dx_s/dt = \dot{\eta}_s \alpha_1 \sqrt{\beta_1}$ using the system (11)-(12) with the velocity of the ablation front in the standard formulation. It consists of solving Eq. (3) with the boundary conditions

$$x = x_a\colon\ T = T_s;\ q = m\rho_i L \frac{dx_a}{dt} - \lambda \frac{\partial T}{\partial x}$$

and the same initial conditions. Using the method presented above it is easy to show that the velocity of the ablation front will be determined by the expression

$$\frac{dx_a}{dt} = q/[m\rho_i L + c\rho (T_s - T_i)]. \qquad (13)$$

As one can see from Fig. 1, the rate of erosion $\dot{x}_a$, found under the condition that there is no transitional zone, is close to $\dot{x}_m$ only in the case when the degree of saturation with water at the front $\sigma_m \simeq 1$. This situation is possible for low flow velocities, when even a small quantity of ice in the pores is sufficient for securing grains of sand. Converserly, for a high flow velocity of the liquid in the well, when the breaking of the ice-cement bonds between the grains of sand becomes possible with a high ice content, which corresponds to small values of σ_m, $\dot{x}_m$ is slightly greater than $\dot{x}_a$. This result is confirmed by the fact that as the velocity of the mud decreases the intensity of the breakup of the walls of the well decreases appreciably. In addition, the inequality $\dot{x}_m > \dot{x}_a$ implies that the volume of the sand carried out of the well should exceed, often by a large amount, the volume of the sand in the thaw zone calculated based on the model of standard ablation. This result is completely confirmed by practical observations [1, 6].

The appreciable decrease, shown in Fig. 1, in the breakup velocity as the initial temperature of the frozen rocks T_i decreases also agrees with experience in drilling wells.

One can see from Fig. 2 that the velocity of the breakup front depends strongly on the intensity of the heat flow to the wall, while the thickness of the intermediate zone is insensitvie to changes in the heat flow. Thus no matter how weakly the well wall is heated by the circulating liquid, a quite appreciable layer of thawed sand forms on the wall. Thanks to this, a lay crust forms on the well wall; this is confirmed by model experiments [1]. The formation of a clay crust cannot be explained based on ablation [2].

The obtained results make it possible to understand the physical basis for the methods, described in [1], used for reducing pocket formation during the drilling of wells using solutions with a positive temperature. First, the velocity of the rising flow of circulating water decreases. In the process, the rate of erosion as well as the intensity of heat transfer decrease. Second, the strength of the bonding of the coagulation structure with the interstitial water increases. The inverse quantity is usually called the water yield or the filtration index of the clay solution. For low water yield the mud itself and not its filtrate is pulled into the interstitital space of the transitional zone freed up as the ice melts. This strengthens the transitional zone. The water yield of the mud can be reduced by introducing polymer additives. These additives can be chosen so as to decrease at the same time the heat transferred from the mud.

NOTATION

Here, x and t are the spatial coordinate and the time; $x_m(t)$, $x_s(t)$ are the coordinates of the erosion and thaw fronts; $x_a(t)$ is the coordinate of the ablation front in the standard formulation; T_i is the initial temperature of the porous medium; T_s is the melting point of the interstitial ice; T_m is the temperature of the soil framework, corresponding to a fixed degree of saturation with water σ_m; $\sigma(x, t)$ is the degree of saturation of the pore space between x_m and x_s with water; $\sigma_m = \sigma(x_m, t)$, λ_w, λ_i, λ_{fw} are the thermal conductivity of ice, water, and the soil framework; $(c\rho)_i$, $(c\rho)_w$, $(c\rho)_{fw}$ are the corresponding volume heat capacities; ρ_i is the density of ice; L is the specific heat of melting of ice; and, m is the porosity of the soil.

LITERATURE CITED

1. R. I. Medvedskii, Construction and Operation of Oil and Gas Wells in Permafrost Rock [in Russian], Moscow (1987).
2. G. S. Gryaznov, Characteristics of Drilling of Deep Wells in Permafrost Regions [in Russian], Moscow (1969).
3. Yu. A. Buevich, Inzh. Fiz. Zh., 54, No. 5, 770-778 (1988).
4. G. I. Barenblatt, Similarity, Self-Similarity, and Intermediate Asymptotic Behavior [in Russian], Moscow (1978).
5. H. Carslaw and D. Jager, Heat Conduction in Solids [Russian translation], Moscow (1964).
6. N. E. Shcherbich, "Investigation of the construction of wells in rocks frozen for many years and recommendations for the prevention of deposits for the example of South-Solenin Deposit," Author's Abstract of Candidate's Dissertation in Technical Sciences, Moscow (1983).

SOLUTION OF THE HEAT CONDUCTION PROBLEM FOR LAMINAR ORTHOTROPIC SLABS IN A SPATIAL FORMULATION

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A solution is obtained for the stationary heat conduction problem in a spatial formulation for rectangular slabs with an arbitrary quantity of orthotropic layers.

A survey of investigations in the area of the analysis of laminar slabs under thermal actions showed that there are no solutions in a three-dimensional formulation for slabs with anisotropic (orthotropic) layers.

A laminar slab in a stationary temperature field is examined in this paper. The slab is referred to a rectangular x_1, x_2, x_3 coordinate system. The problem is solved by the conjugatuion method [1]. Zero temperature is maintained on the slab side surfaces, i.e., we have T = 0 for $x_1 = 0, a_1$ and $x_2 = 0, a_2$. The following boundary conditions [2] are possible on the slab face surfaces ($x_3 = b^{(\ell)}$, $\ell = 0, n$):

1) First kind

$$T(x_i, b^{(l)}) = f^{(l)}(x_i),\ l = 0,\ n;\ i = 1,\ 2; \quad (1)$$

2) Second kind

$$\lambda_3^{(1)} T_{,3}(x_i, b^{(0)}) = q^{(0)}(x_i);\ \ \lambda_3^{(n)} T_{,3}(x_i, b^{(n)}) = q^{(n)}(x_i); \quad (2)$$

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